feb/20/20

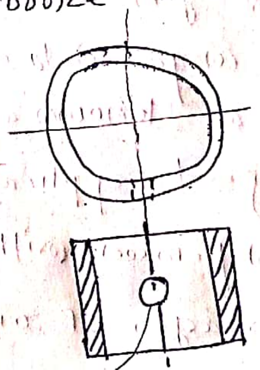
Bearing Constructions :-

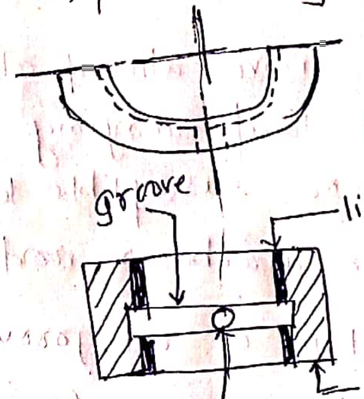
- There are two types of bearing construction
  - Solid bushing
  - lined bushing

* Solid Bushing :-

- made either by casting or machining from a bar
- Finishing by grinding and reaming operation.

ex :- bronze bearing





* Lined Bushing :-

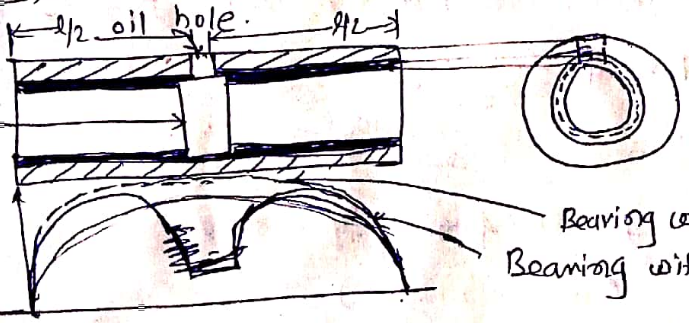
- steel backing with a thin lining of bearing material like babbitt
- generally split into two half.

* Patterns of oil Grooves - 2 types (circumferential & cylindrical)

1) Circumferential oil grooves

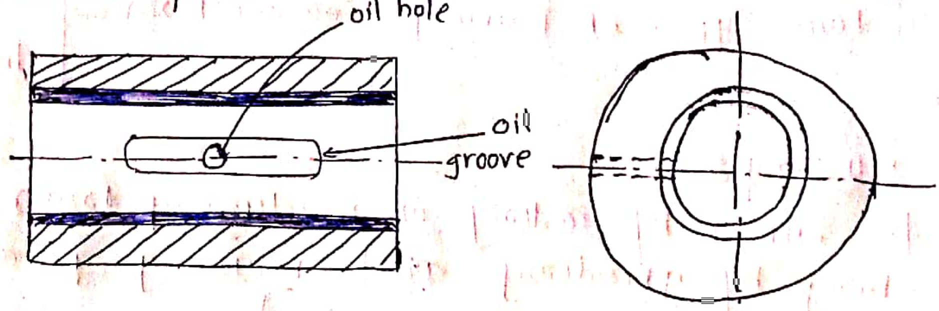
- divide bearing into two short bearing
- pressure developed along axis is reduced due to groove result in lower load carrying capacity
- ~~circumferen~~ Centrifugal force acting on oil in circumferential groove may build higher pressure than supply pressure which may restrict the flow of lubricant.

Application :- connecting rod & crankshaft of automotive engine



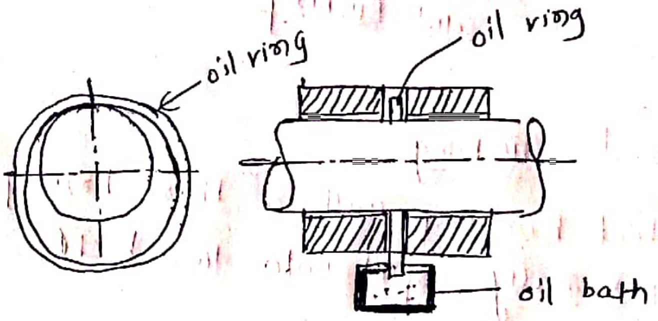
Circumferential oil Groove

② Cylindrical Oil Groove :-



- It has axial groove almost along the length of bearing
- It has higher load carrying capacity compared to circumferential but it is more susceptible to vibration known as oil whip.
- It is used for gearboxes and high speed application
- Different patterns of oil groove are obtained with combination of cylindrical & circumferential passages of oil

Oil Ring Bearing :-



- It consist of oil ring in contact with shaft and dipping in oil bath below.
- Diameter of oil ring is large compared with diameter of shaft.
- As shaft rotates, oil ring rotates but with lower speed & carries oil along with it from oil bath to shaft
- There are spreader grooves, which carry oil to entire surface of shaft.
- Bearing use is restricted to horizontal shaft.

White metal contains, tin, lead, cadmium, bismuth, zinc., antimony

lead based -

White Metal (Tin based, lead based)

Tin based Tin (90-92 %) Sb (Antimony 4.5-5.5 %), Cu (Copper 3.5-4.5 %)
Pb (lead - 0.2 % max.)

Bearing Materials :-

Desirable properties of bearing materials

i) Bearing material should not damage the surface of journal, it should not stick or weld to journal surface

ii) high compressive strength to withstand high pressure without distortion.

iii) High Endurance strength:

In application like connecting rod, it is subjected to fluctuating stresses so in order to avoid failure due to pitting, sufficient endurance strength should be there.

iv) Conformability:-

- ability to yield and adopt its shape to that of journal so due to any load, bearing will adopt itself to ~~journa~~ deflected journal shaft.

v) Embeddability:-

- Embed dirt particles and avoid scratches on surface of journal
- bearing material should be soft to allow these particles to get embeded in lining & avoid trouble.

v) Corrosion Resistance:-

- Bearing material should have sufficient corrosion resistance so oxidation cause by excessive temp. of lubricating oil which form the corrosive acid can be ~~avoi~~ ~~avo~~ taken care

vi) Resonable cost & easily available

Most popular material

① Babbitt / white metal [Antimony, tin, lead, cadmium, bismuth, zinc]

- silvery appearance
- lead based or Tin based depending upon major alloying element
- used in the form of strip or thin lining (about 0.5 mm thick) bonded to steel shell.

- excellent conformability & embeddability.
- Tin-based babbit → excellent corrosion resistance & can be easily bonded to steel.
  → high cost and shortage of tin — limitation.

- Babbitt with lead base or tin base are inherently weak and their strength decreses rapidly with increasing temp.
- their use is further restricted due to poor fatigue strength.
- 10 Grades of White metals (90, 84, 75, 69, 60, 20, 10, 6, 5)

② Bronze, Copper-lead, Aluminium alloy & plastic

② - bronze is cheaper compared to babbitt, stronger & can withstand higher pressure.
it has excellent casting & maching characteristics
- made in single solid unit.
- limitation → tendency to stick to surface of journal at higher temp.

③ Copper-lead bearing (70% Cu & 30% Pb)

- used in thin lining like white metal
- more hardness & fatigue strength & used in heavy duty application at high temp.
- stick

④ Tin-Aluminium alloy :-

- Higher fatigue strength & retain their strength at high temp.
- cost - limitation (tin)

⑤ Non-metallic :- Graphite, Plastic (Teflon) & Rubber

- for high temp. application - graphite.
- Teflon - extremely low coeff. of friction & lubricant required
  - Bearing located at inaccessible position.
- these bearing used.
  - food processing to avoid contamination of oil

Rubber :- Marine application

* Sintered Metal Bearing :-

- sintered metal powder ~~as~~ bearing made from compressed metal powder by sintering process

- porous & impregnated with lubricating oil
- absorb lubricating oil to extent of 20-30% of their vol.
- enabling bearing to run for longer period without any attention.
- periodically lubricated.

Two Grades

Copper based	Iron based
more corrosive resistance.	
$p = 60\ N/mm^2$	$p = 100\ N/mm^2$

- satisfactory performance up to 80° C.

Application - automobiles, textile, machinery, m/c tool.

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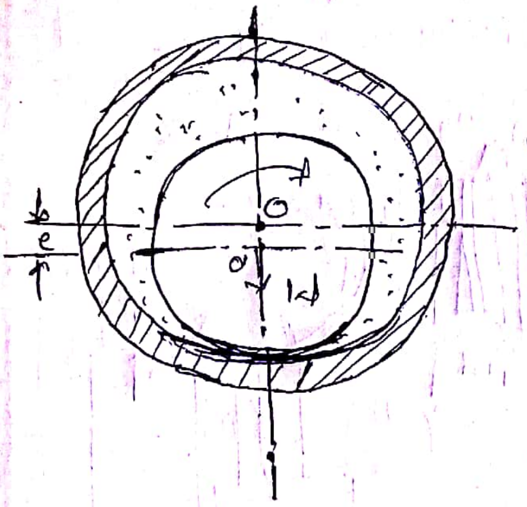
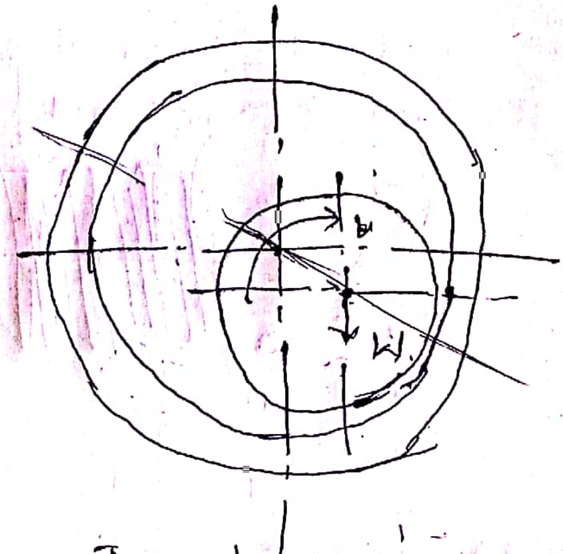
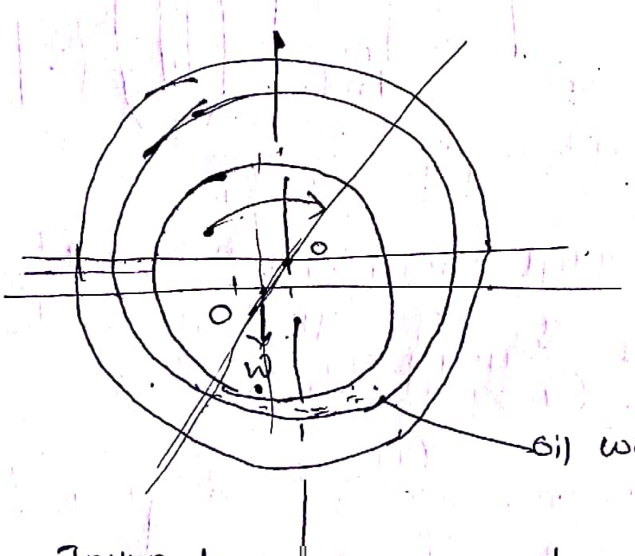
Hydrodynamic lubrication



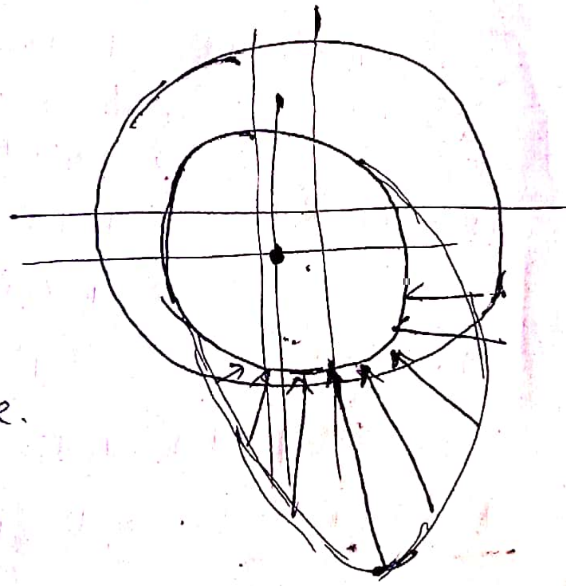
Journal at rest



Journal start to rotate



Journal at full speed



Pressure Distribution in Hydrodynamic Bearing.

Lubrication :-

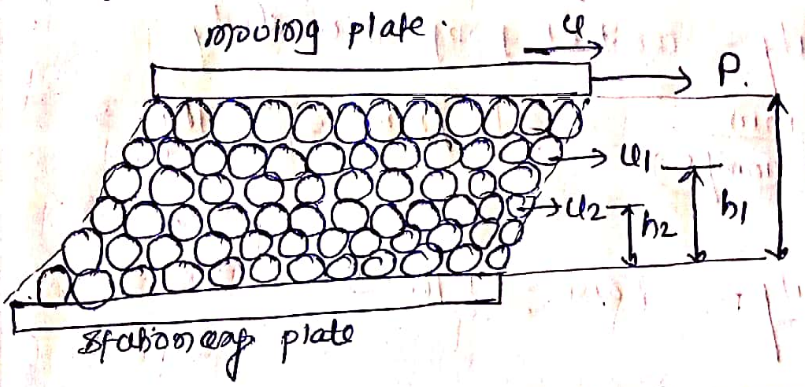
It is a science of reducing friction by the application of suitable substance called lubricant, between the rubbing surfaces of bodies having relative motion.

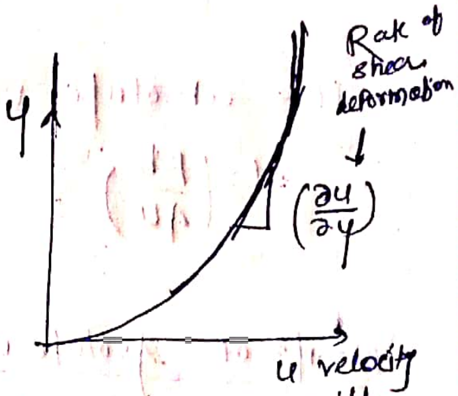
- Lubricants are classified into three groups
  i) Liquid lubricants → like vegetable or mineral oils
  ii) Semi-solid lubricants → grease
  iii) Solid lubricants → graphite, Molybdenum disulphide

- objectives of lubrication :-
  i) to reduce friction
  ii) to prevent wear
  iii) to carry away heat generated due to friction
  iv) to protect journal & bearing from corrosion.

Viscosity :-

- Viscosity is defined as the internal frictional ~~force~~ resistance offered by a fluid to change its shape or relative motion of its parts.
- An oil film placed betn two parallel plates is shown in Fig.





- Lower plate is stationary & upper plate is moving with velocity by means of force P.
- molecules which mills in layer betn two plates.
- Oil stick to both plates, layers in contact with stationary plate has zero velocity similarly layer in contact with upper plate move with a velocity U.
- The intermediate layers will move with velocities which are proportional to their distance from stationary plate.

$$\frac{U}{h} = \frac{U_1}{h_1} = \frac{U_2}{h_2}$$

- This type of orderly movement is called streamline laminar or viscous flow.
- Tangential force which is acting over layer is shear stress. $\tau = (P/A)$ while a ratio (U/h) is the rate of shear ~~strain~~
- According to newton's law of viscosity, the shear stress is proportional to the rate of shear at any point in the fluid, therefore

$$(P/A) \propto (U/h)$$

$$\boxed{P = \mu A\left(\frac{U}{h}\right)}$$

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- When the velocity distribution is non-linear with respect to h, the term (U/h) in the above equation is replaced by (dU/dh) & eqn can be rewritten as

$$P = \mu A \left(\frac{dU}{dh}\right)$$

μ = absolute viscosity = const. of proportionality

$$\mu = \left(\frac{Ph}{AU}\right) = \frac{N\ mm}{mm^2\ (mm/s)} = Ns/mm^2 = MPa-s$$

<u>Units of Absolute viscosity</u> :- Poise, MPa-S

Dyne-s/cm²

N-s/mm²

Poise = dyne-s/cm²

Centi Poise (CP)

$$1\,cP = \frac{1}{10^2}\ \text{poise} = 10^{-2}\ \text{poise}$$

$$= 10^{-9}\ Ns/mm^2$$

$$1\ Ns/mm^2 = 1\,MPa\text{-}s = 10^9\ cP$$

$$\mu = \frac{Z}{10^9}$$

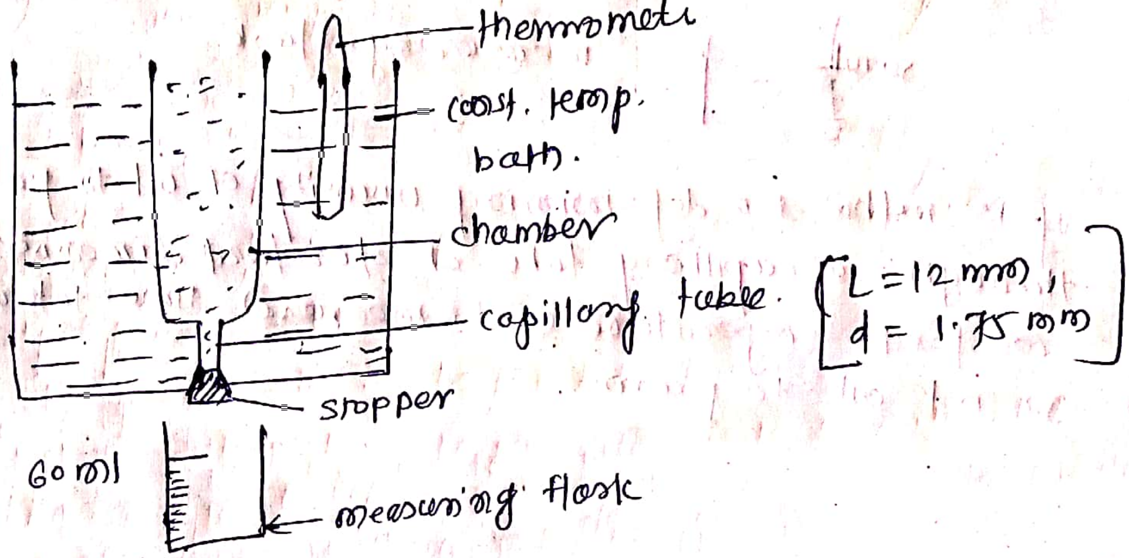
Z = viscosity in centiPoise

μ = viscosity in MPa/S, Ns/mm²

* Measurement of viscosity :-

- difficult to measure viscosity with two plates experiment
- popular method for determining viscosity are
i) Saybolt Universal viscometer (USA)
ii) Redwood viscometer (UK)
iii) Engler viscometer (Indian sub continent)

i) Saybolt Universal viscometer (USA)



- Viscosity is measure by passing liquid through a capillary tube of standard dimension & time taken for 60 ml of liquid is measured. (s)
- Oil whose viscosity is to be measured is immersed in constant temp. bath.
- Unit of viscosity is saybolt universal second which is related to kinematic viscosity by following relationship

$$Z_k = \left[0.22t - \frac{180}{t}\right]$$

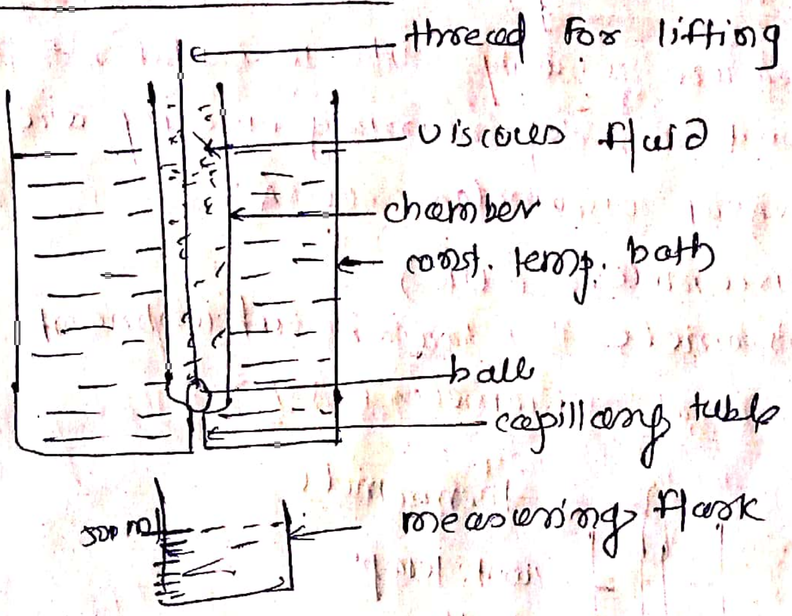
t = time in s.
Z_k = kinematic viscosity in centistokes.

* Kinematic viscosity :- is defined as the ratio of absolute viscosity to density of lubricant

$$\boxed{Z_k = \frac{Z}{\rho}}$$

ρ = density of lubricant in g/cm³

② Redwood Viscometer :-



- It is method for determining viscosity of oil to pass through a capillary tube of standard dimension at constant temperature & to measure time taken for 50 ml oil to pass.

③ Engler :-

In Engler viscometer, the viscosity is measured in terms of Engler degree (°E), which is the ratio of time taken by the oil to the time taken by water at same temp.

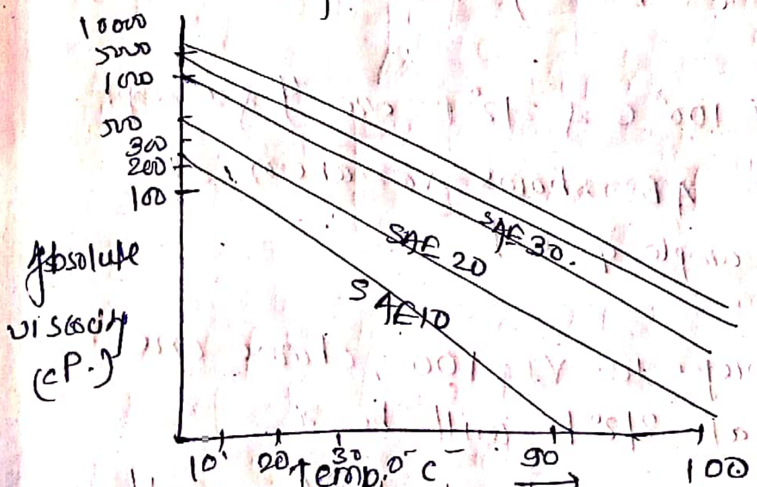
* Viscosity Index :-

- The viscous resistance of lubricating oil is due to intermolecular forces.
- As temp. increases, the oil expands and the molecules move further apart, decreasing the intermolecular force in consequence.
- Viscosity of lubricating oil decreases with increasing temp.
- Approximate relation betn viscosity & temp. is as follows

$$\log \mu = A + (B/T)$$

A & B = constant

T = Absolute temp.



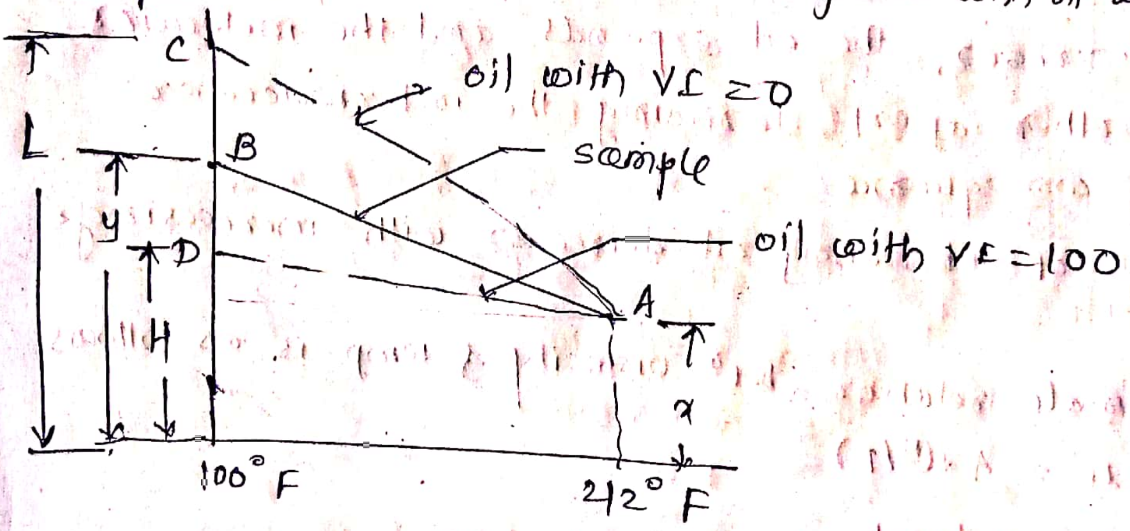
- The rate of change of viscosity with respect to temp. is indicated by a no. called viscosity index. (VI)
- Viscosity Index is defined as an arbitrary no. used to characterize the variation of kinematic viscosity of lubricating oil with temp.
- In order to find out viscosity index of the oil, two groups of reference oils are considered.

i) VI = 100 → These oil have very small change in viscosity with temp.

ii) VI = 0 → these oil have very large change in viscosity with temp.

- given oils are compared with these two references
- oil with VI=70 has less rate of change of viscosity with ~~ref~~ temp. compared with oil with VI=60.

Steps :-



① Measure viscosity of given sample ~~of~~ oil at 100° F & 212 °F

- suppose viscosity at 100° C & 212° F are y & x resp.
- Plot x & y & line AB shows variation of viscosity for given sample of oil

② Among first ref. group ie. VI=100, choose one oil whose viscosity at 212° F will be x,

- find the viscosity of chosen oil at 100°C ie. H.
- Draw DA which shows variation of viscosity for ref. oil with VI = 100.

③ Among second ref. group ie. VI=0, choose one oil whose viscosity at 212° F will be x.

- find viscosity of chosen oil at 100°C ie. C
- Draw CA which shows variation of viscosity for ref. oil
- According to ASTM standards, the viscosity index is given by

$$V.I = \left(\frac{L-y}{L-H}\right) \times 100 \%$$

Mckee's Investigation :-

- In hydrodynamic bearings, initially the journal is at rest, there is no relative motion & no hydrodynamic film so there is metal to metal contact between surface of journal & bearing.

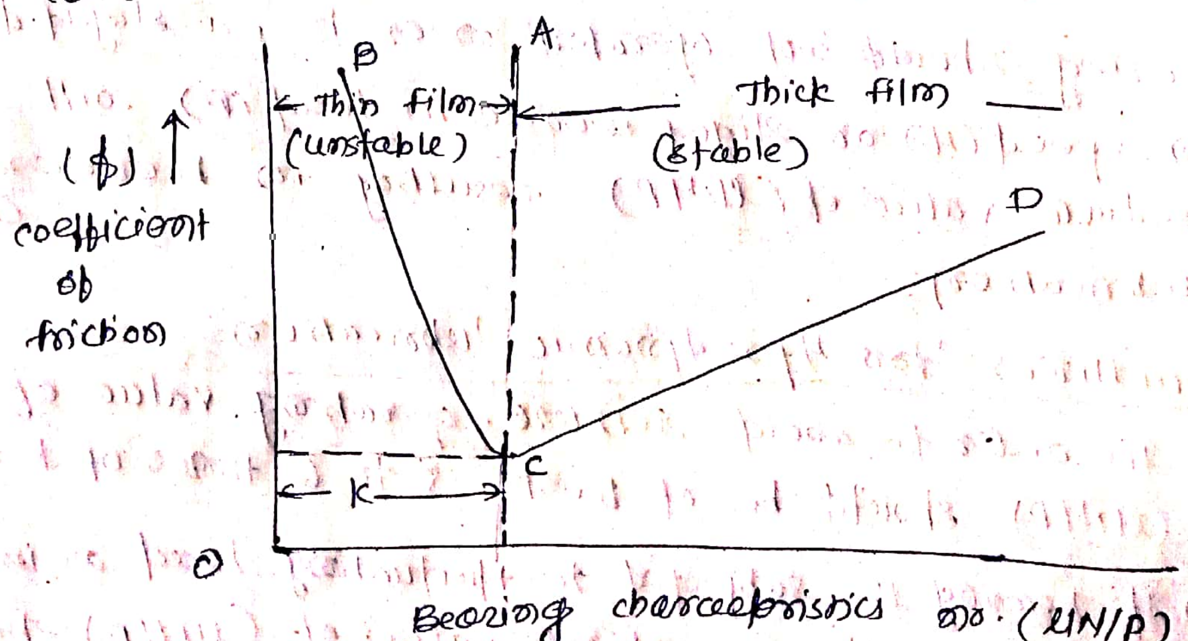
- As journal start to rotate, it takes some time for the hydrodynamic film to built sufficient pressure in clearance space

- During this period, there is partial metal to metal contact & partial lubricant film. This is thin film lubrication.

- As speed is increased, more & more lubricant is forced into wedge-shape clearance space & sufficient pressure is built up, separating surfaces of journal & bearing. This is called thick film lubrication.

- There is a transition from thin film lubrication to thick film lubrication as speed increases.

- The transition can be better visualized by (μN/P) curve



μ = absolute viscosity of lubricant
N = speed of journal
P = unit ~~press~~ bearing pressure (load per unit of projected area of bearing)

- This curve is an experimental curve developed by McKee.
- Bearing characteristic no. is dimensionless group of parameter.
- Bearing characteristics no. is plotted on x-axis & coeff. of friction f is plotted on ordinate.
- Coefficient of friction :- is the ratio of tangential friction force to the radial load acting on the bearing.

(i) In region BC, there is partial metal to metal contact & partial patches of lubricant. This is the condition of thin film or boundary lubrication.

(ii) In region CD, there is relatively thick film of lubricant & hydrodynamic lubrication takes place.

(iii) AC is dividing line betn these two mode.

(iv) Coefficient of friction is minimum at C or at transition betn two mode. The value of bearing characteristics no. corresponding to this minimum coeff. is called bearing modulus (K)

- Bearing should not operated near K, a slight drop in speed (N) or slight increase in load (P), will reduce value of (μN/P) resulting in boundary lubrication.

* Guidlines for Hydrodynamic lubrication

i) In order to avoid seizure, operating value of (μN/P) should be at least 5 to 6 times of K.

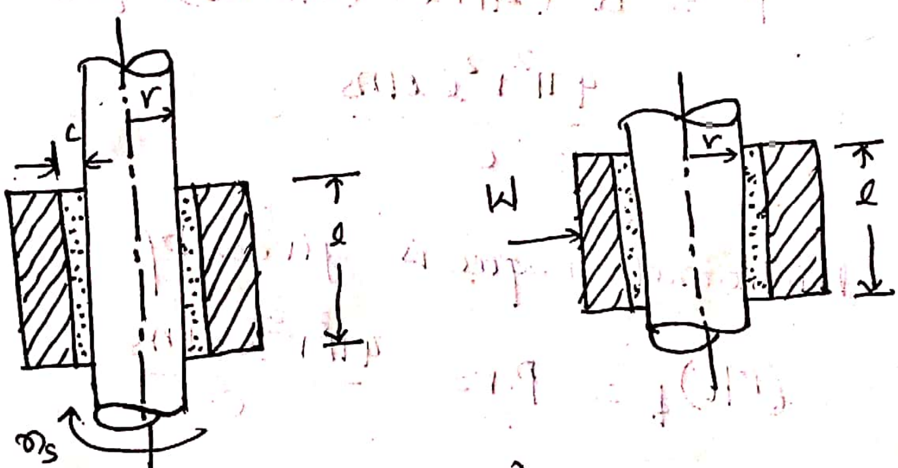
ii) If bearing is subjected to fluctuating load or impact conditions, the operating value of (μN/P) should be 15 times of K.

- For low viscosity (μ), (μN/P) is low & boundary lubrication
- (μN/P) defines the stability of hydrodynamic journal bearing

* Petroff's Equation :-

- Petroff's equation is used to determine the coefficient of friction in journal bearings :-
- Assumptions
  i) shaft is concentric with bearing.
  ii) bearing is subjected to light load.

- This equation defines the group of dimensionless parameters that govern the frictional properties of the bearing.



r = radius of the journal (mm)
l = length of bearing (mm)
c = radial clearance
n_s = journal speed (rev/s)

- The velocity at the surface of the journal is given by

$$U = r\omega = r.(2\pi n_s) \quad \text{——} \quad (1)$$

- Newton's law of viscosity

$$P = \mu A\left(\frac{U}{h}\right) \quad \text{——} \quad (2)$$

- We will apply this eqn for viscous flow through the annular portion betn journal & bearing in circumferential directn

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P = tangential frictional force

A = Area of journal surface $(2\pi r)l$.

U = surface velocity = $(2\pi r)n_s$

h = dist^n bet^n journal & bearing surface = c

Substituting above values in eq^n (2)

- Tangential frictional force is

$$P = \mu (2\pi r l)(2\pi r n_s)\frac{1}{c}$$

$$= \frac{4\pi^2 r^2 l \mu n_s}{c} \quad \text{--- (3)}$$

- Frictional torque is given by

$$(M_t)_f = P.r = \frac{4\pi^2 r^3 l \mu n_s}{c} \quad \text{--- (4)}$$

- Let us consider the radial force W acting on the bearing.

- The unit pressure (p) acting on the bearing is given by.

$$p = \frac{W}{\text{projected Area of bearing}} = \frac{W}{(2rl)}$$

$$W = 2prl$$

- The frictional force will be $(f W)$ & frictional torque will be $(f W r)$

$$(M_t)_f = f Wr = f(2prl)r = f(2pr^2 l) \quad \text{--- (5)}$$

comparing (4) & (5)

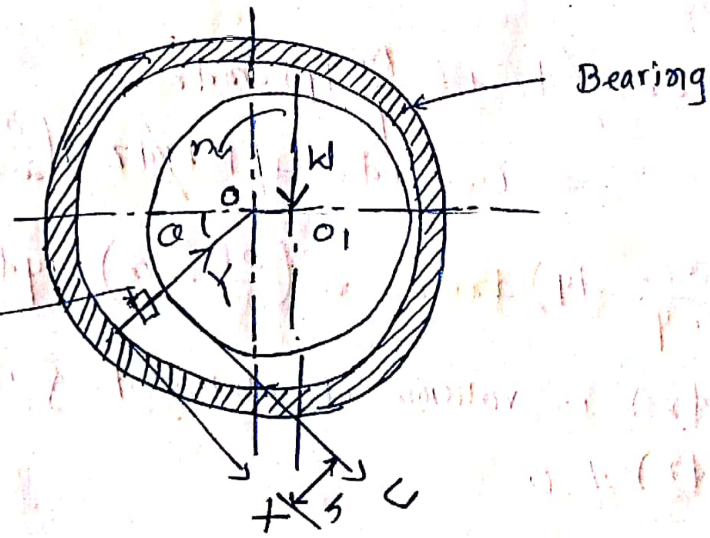
$$\frac{4\pi^2 r^3 l \mu n_s}{c} = f(2pr^2 l)$$

$$\boxed{f = 2\pi^2 \left(\frac{r}{c}\right)\left(\frac{\mu n_s}{P}\right)}$$ — Petroff's Equation.

$\left(\frac{r}{c}\right)$ & $\left(\frac{\mu n_s}{P}\right) \longrightarrow$ dimensionless parameters that governs the coeff. of ~~frictional properti~~ friction & other frictional properties.

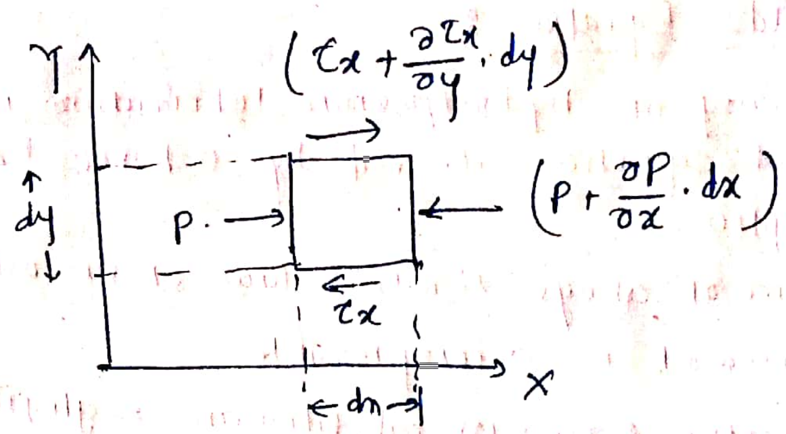
Reynold's Equation :-

- The theory of hydrodynamic lubrication is based on differential equation derived by Osborne Reynold.
- Assumption
  i) lubricant obeys Newton's law of viscosity.
  ii) lubricant is incompressible
  iii) inertia forces in oil film are negligible
  iv) Viscosity of lubricant is constant
  v) Effect of curvature of film with respect to film thickness is ~~negligible~~ neglected ie. it is assumed that film is so thin that the pressure is constant across the film thickness.
  vi) Both shaft & bearing are rigid.
  vii) There is continuous supply of lubricant.



Fluid Element in X-Y Plane.

- An element having dimensions dx, dy & dz is considered in this analysis
- X- axis in the direction of motion.
  Y- axis in the radial plane
  Z → axis parallel to the axis of the journal
  u, v & w → velocities in x, y & z-direction.
  τ_x & τ_z → shear stresses along X & Z direction
  p = fluid film pressure



Equilibrium of forces in X- direction

– The forces acting on the element in x-direction are shown in fig. considering equilibrium of forces,

$$p \cdot dy\,dz + \left(\tau_x + \frac{\partial \tau_x}{\partial y} \cdot dy\right) dx\,dz \qquad \text{—— (1)}$$

$$= \tau_x \cdot dx\,dz + \left(p + \frac{\partial p}{\partial x} \cdot dx\right) dy \cdot dz$$

$$\cancel{p\,dy\,dz} + \cancel{\tau_x \cdot dx\,dz} + \left(\frac{\partial \tau_x}{\partial y}\, dy\right) dx\,dz$$

$$= \cancel{\tau_x\,dx\,dz} + \cancel{p\,dy\,dz} + \left(\frac{\partial p}{\partial x} \cdot dx\right) dy\,dz$$

$$\left(\frac{\partial \tau_x}{\partial y} \cdot dy\right) dx\,dz = \left(\frac{\partial p}{\partial x} \cdot dx\right) dy\,dz$$

The $(dx \cdot dy \cdot dz)$ is volume of element & it is +ve value $(dx\,dy\,dz) \neq 0$.

$$\frac{\partial \tau_x}{\partial y} = \frac{\partial p}{\partial x} \qquad \text{—— (2)}$$

According to Newtons law of viscosity

$$\tau_x = \mu \cdot \frac{\partial u}{\partial y} \qquad \text{—— (3)}$$

$$\frac{\partial \tau_x}{\partial y} = \mu \cdot \frac{\partial^2 u}{\partial y^2}$$

From eqn (2) & (3)

$$\frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu}\frac{\partial p}{\partial x}.$$

Integrating above equation with y twice gives

1st $\int \frac{\partial^2 u}{\partial y^2} dy = \frac{1}{\mu}\int \frac{\partial p}{\partial x} dy$

$$\frac{\partial u}{\partial y} = \frac{1}{\mu}\frac{\partial p}{\partial x} y + C_1$$

2nd

$$\int \frac{\partial u}{\partial y} \cdot dy = \int \frac{1}{\mu}\frac{\partial p}{\partial x} y \, dy + \int C_1 \, dy$$

$$u = \frac{1}{\mu}\frac{\partial p}{\partial x}\cdot\frac{y^2}{2} + C_1 y + C_2 \quad \text{—— (4)}$$

C_1 & C_2 are constant of integration & are evaluated from following two boundary conditions

At $y=0$, $u=0$

At $y=h$, $u=U$

using 1st B.C. at $y=0$, $u=0$ in (4)

$$0 = 0 + C_1 \times 0 + C_2 \Rightarrow C_2 = 0$$

using 2nd BC. at $y=h$ $u=U$ in (4)

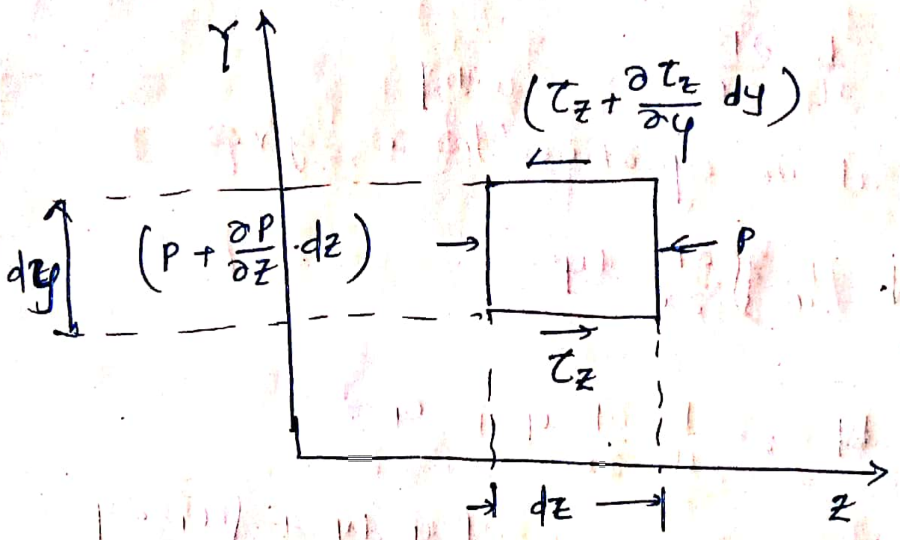
$$U = \frac{1}{\mu}\frac{\partial p}{\partial x}\cdot\frac{h^2}{2} + C_1 h + 0$$

$$C_1 = \frac{U}{h} - \frac{1}{\mu}\frac{\partial p}{\partial x}\cdot\frac{h}{2} \quad \& \quad C_2 = 0 \quad \text{—— (5)}$$

substituting C_1 & C_2 in equation (4)

$$u = \frac{1}{\mu}\frac{\partial p}{\partial x}\cdot\frac{y^2}{2} + \frac{Uy}{h} - \frac{1}{\mu}\frac{\partial p}{\partial x}\cdot\frac{h}{2} y + 0$$

$$\boxed{u = \frac{Uy}{h} + \frac{1}{2\mu}\frac{\partial p}{\partial x}(y^2 - hy)} \quad \text{—— (6)}$$



Equilibrium of forces in z-direction

- The forces acting on the element in z-direction are shown in fig. Considering equilibrium of forces in z-direction.

$$p \cdot dy \cdot dx + \left(\tau_z + \frac{\partial \tau_z}{\partial y} \cdot dy\right) \cdot dz \cdot dx = \tau_z \cdot dz \cdot dx + \left(p + \frac{\partial p}{\partial z} \cdot dz\right) dy \cdot dx \quad \text{—— (7)}$$

$$p\, dx\, dy + \tau_z \cdot dx\, dz + \left(\frac{\partial \tau_z}{\partial y} \cdot dy\right) dx\, dz = \tau_z \cdot dx\, dz + p\, dx\, dy + \frac{\partial p}{\partial z} dz \cdot dx\, dy$$

As $(dx\, dy\, dz) \neq 0$

$$\frac{\partial \tau_z}{\partial y} = \frac{\partial p}{\partial z} \quad \text{—— (8)}$$

According to Newton's law of viscosity

$$\left.\begin{aligned} \tau_z &= \mu \cdot \frac{\partial w}{\partial y} \\ \frac{\partial \tau_z}{\partial y} &= \mu \cdot \frac{\partial^2 w}{\partial y^2} \end{aligned}\right\} \quad \text{—— (9)}$$

From equation (8) & (9)

$$\frac{\partial^2 w}{\partial y^2} = \frac{1}{\mu} \cdot \frac{\partial p}{\partial z}.$$

Integrating above equation twice gives w.r.t. y gives

$$\int \frac{\partial^2 w}{\partial y^2} \cdot dy = \int \frac{1}{\mu} \frac{\partial P}{\partial z} \cdot dy$$

$$\frac{\partial w}{\partial y} = \frac{1}{\mu} \cdot \frac{\partial P}{\partial z} \cdot y + C_3$$

$$\int \frac{\partial w}{\partial y} dy = \int \left[\frac{1}{\mu} \frac{\partial P}{\partial z} \cdot y + C_3 \right] dy$$

$$w = \frac{1}{\mu} \cdot \frac{\partial P}{\partial z} \cdot \frac{y^2}{2} + C_3 y + C_4 \qquad \text{—— (10)}$$

C_3 & C_4 are constant of integration & are obtained by boundary conditions

BC-① $w = 0$ when $y = 0$

BC-② $w = 0$ when $y = h$

Substituting BC-① in ⑩ gives.

$$0 = 0 + C_3 \times 0 + C_4 \Rightarrow C_4 = 0$$

Substituting BC-② in ⑩ gives

$$0 = \frac{1}{\mu} \cdot \frac{\partial P}{\partial z} \cdot \frac{h^2}{2} + C_3 h + 0$$

$$\left[C_3 = -\frac{1}{\mu} \cdot \frac{\partial P}{\partial z} \cdot \frac{h}{2} \right] \; \& \; C_4 = 0 \qquad \text{—— (11)}$$

Substituting C_3 & C_4 in ⑩ gives

$$w = \frac{1}{\mu} \cdot \frac{\partial P}{\partial z} \cdot \frac{y^2}{2} + \left(-\frac{1}{\mu} \frac{\partial P}{\partial z} \cdot \frac{h}{2} \right] y + 0$$

$$\boxed{w = \frac{1}{2\mu} \frac{\partial P}{\partial z} (y^2 - hy)} \qquad \text{—— (12)}$$

- The general continuity equation for incompressible flow is given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad \text{——— (13)}$$

- Despite there is no flow in Y-direction, the local continuity equation in three direction must be satisfied so.

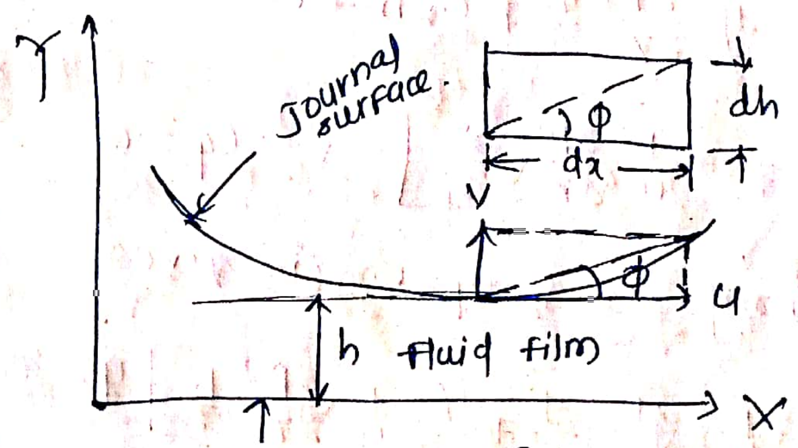
$$\frac{\partial v}{\partial y} = -\frac{\partial u}{\partial x} - \frac{\partial w}{\partial z}$$

- Integrating the above equation w.r.to. y within limits 0 to h

$$\int_0^h \frac{\partial v}{\partial y} dy = -\int_0^h \frac{\partial u}{\partial x} \cdot dy - \int_0^h \frac{\partial w}{\partial z} \cdot dy \quad \text{——— (14)}$$

- The left side of equation can be expressed as

$$\int_0^h \frac{\partial v}{\partial y} \cdot dy = \left(V_{at\ y=h} - V_{at\ y=0} \right) \quad \text{——— (15)}$$



Above figure shows the fluid film in X-Y plane. when (y=0), it indicates stationary bearing surface & in Y-direction v is zero.

When $(y=h)$ it indicates journal surface & velocity in Y direction (v) is given by

$$\tan\phi = \frac{V}{U} = \frac{dh}{dx}$$

$$\therefore \quad V = U \cdot \frac{dh}{dx}.$$

In the above expression curvature effect is neglected substituting the value of v in (15).

$$\int_0^h \frac{\partial v}{\partial y}\, dy = \left(V_{at\ y=h} - V_{at\ y=0}\right)$$

$$\int_0^h \frac{\partial v}{\partial y} \cdot dy = U\frac{dh}{dx} - 0 = U\frac{dh}{dx} \quad \text{——} \quad (16)$$

from equation (14) & (16)

$$\int_0^h \frac{\partial u}{\partial x} \cdot dy + \int_0^h \frac{\partial w}{\partial z} \cdot dy = -U\frac{dh}{dx}. \quad \text{——} \quad (17)$$

Now, we will apply leibnitz theorem for interchanging the signs of integration and differentiation of the first term of the above equation, because the upper limit 'h' is a function of x.

According to leibnitz theorem

$$\frac{d}{dx}\int_{h_1(x)}^{h_2(x)} \frac{\partial u}{\partial x} \cdot dy = \frac{d}{dx}\int_{h_1(x)}^{h_2(x)} u(x,y)\, dy$$

$$= \int_{h_1(x)}^{h_2(x)} \frac{\partial}{\partial x} u(x,y)\, dy + \left[u[h_2(x), x]\frac{d h_2(x)}{dx}\right.$$

$$\left. - u[h_1(x), x)\frac{d h_1(x)}{dx}\right]$$

Substituting following values

$h_1(x) = 0$

$h_2(x) = h$

$u(x,y) = u$

$$u[h_1(x), x] = u \text{ at } [h_1(x), x] = 0$$

$$u[h_2(x), x] = u \text{ at } [h_2(x), x] = U$$

We get

$$\frac{d}{dx}\int_0^h u\,dy = \int_0^h \frac{\partial u}{\partial x}\,dy + U\frac{dh}{dx} + 0$$

- First term of equation (17) is given by

$$\int_0^h \frac{\partial u}{\partial x}\,dy = \frac{\partial}{\partial x}\int_0^h u\,dy - U\frac{dh}{dx}. \qquad \text{—— (18)}$$

- In the second term of equation (17), the upper limit h is constant with respect to y or z.
- Therefore the sign of integration & differentiation can be interchanged

$$\int_0^h \frac{\partial w}{\partial z}\,dy = \frac{\partial}{\partial z}\int_0^h w\,dy \qquad \text{—— (19)}$$

- Substituting (18) & (19) in equation (17) gives

$$\frac{\partial}{\partial x}\int_0^h u\,dy - \cancel{U\frac{dh}{dx}} + \frac{\partial}{\partial z}\int_0^h w\,dy = -\cancel{U\frac{dh}{dx}}$$

$$\boxed{\frac{\partial}{\partial x}\int_0^h u\,dy + \frac{\partial}{\partial z}\int_0^h w\,dy = 0} \qquad \text{—— (20)}$$

Substituting the values of u & w from eqn. (6) & (12) gives

$$\frac{\partial}{\partial x}\int_0^h \left[\frac{U}{h}y + \frac{1}{2\mu}\frac{\partial p}{\partial x}(y^2 - hy)\right]dy \qquad (21)$$

$$+ \frac{\partial}{\partial z}\int_0^h \left[\frac{1}{2\mu}\frac{\partial p}{\partial z}(y^2 - hy)\right]dy = 0$$

1st term :-

$$\frac{\partial}{\partial x}\int_0^h \left[\frac{U}{h}y + \frac{1}{2\mu}\frac{\partial p}{\partial x}\cdot(y^2 - hy)\right]dy$$

$$= \frac{\partial}{\partial x}\int_0^h \left[\frac{Uy}{h} + \frac{1}{2\mu}\frac{\partial p}{\partial x}\cdot(y^2 - hy)\right]dy$$

$$= \frac{\partial}{\partial x}\left[\frac{Uy^2}{2h} + \frac{1}{2\mu}\cdot\frac{\partial p}{\partial x}\cdot\left(\frac{y^3}{3} - \frac{hy^2}{2}\right)\right]_0^h$$

$$= \frac{\partial}{\partial x}\left[\frac{U(h^2 - 0^2)}{2h}\right) + \frac{1}{2\mu}\frac{\partial p}{\partial x}\left(\frac{h^3 - 0^3}{3} - \frac{h}{2}(h^2 - 0)\right)\Bigg]$$

$$= \frac{\partial}{\partial x}\left[\frac{Uh}{2} + \frac{1}{2\mu}\frac{\partial p}{\partial x}\cdot\left(-h^3/6\right)\right]$$

$$= \frac{U}{2}\frac{\partial h}{\partial x} - \frac{1}{12\mu}\frac{\partial}{\partial x}\left[h^3\frac{\partial p}{\partial x}\right] \qquad (22)$$

2nd term :-

$$\frac{\partial}{\partial z}\int_0^h w\,dy = \frac{\partial}{\partial z}\int_0^h \left[\frac{1}{2\mu}\frac{\partial p}{\partial z}(y^2 - hy)\right]dy$$

$$= \frac{\partial}{\partial z}\left[\frac{1}{2\mu}\frac{\partial p}{\partial z}\cdot\left(\frac{y^3}{3} - \frac{hy^2}{2}\right)\right]_0^h$$

$$= \frac{\partial}{\partial z}\left[\frac{1}{2\mu}\cdot\frac{\partial p}{\partial z}\cdot\left(-h^3/6\right)\right]$$

$$\frac{\partial}{\partial z}\int_0^h w\,dy = -\frac{1}{12\mu}\frac{\partial}{\partial z}\left[h^3\frac{\partial p}{\partial z}\right] \quad \text{—— (23)}$$

- substituting equation (22) & (23) in equation 21

$$\frac{U}{2}\frac{\partial h}{\partial x} - \frac{1}{12\mu}\frac{\partial}{\partial x}\left[h^3\frac{\partial p}{\partial x}\right] - \frac{1}{12\mu}\frac{\partial}{\partial z}\left[h^3\frac{\partial p}{\partial z}\right] = 0$$

$$\boxed{\frac{\partial}{\partial x}\left[h^3\frac{\partial p}{\partial x}\right] + \frac{\partial}{\partial z}\left[h^3\frac{\partial p}{\partial z}\right] = 6\mu U\left(\frac{\partial h}{\partial x}\right)}$$

- The above equation is known as Reynold's equation. There is no exact analytical solution for this equation for bearing with finite length.
- Theoretically, exact solution can be obtained if the bearing is assumed to be either infinitely long or very short. This two solutions are called Sommerfeld's solutions.
- Approximate solutions using numerical methods for bearing with finite length are available.

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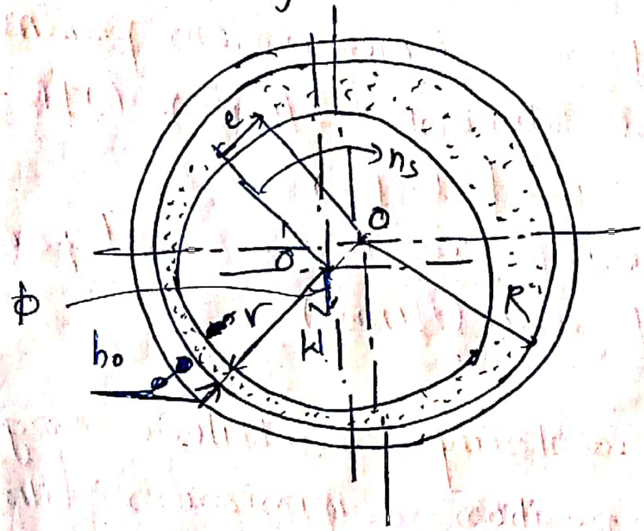
* Raimondi and Boyd Method :-

- There is no exact solution to Reynold's eqn. Raimond and Boyd solved using computational techniques & expressed performance of bearing in terms of dimensionless parameters.



- O & O' are the axes of bearing & journal respectively.

OO' distance = e = eccentricity

(mm) Radial clearance $c = R - r$

R = radius of bearing (mm)
r = radius of journal (mm)
h_o = minimum film thickness (mm)

* Eccentricity ratio = ratio of eccentricity to radial clearance

$$\varepsilon = \frac{e}{c} = \text{Eccentricity ratio} \qquad e = \varepsilon c$$

from figure

$$R = r + e + h_o$$

$$c = R - r = (r + e + h_o) - r = e + h_o = \varepsilon c + h_o$$

$$h_o = c - \varepsilon c = c(1-\varepsilon)$$

$$\varepsilon = 1 - (h_o/c)$$

- The quantity (h_0/c) is called minimum film thickness variable

- The sommerfeld number is given by

$$S = (r/c)^2 \left(\frac{\mu n_s}{p} \right)$$

S = sommerfeld no. (dimensionless parameter)
μ = viscosity of lubricant ($N s/mm^2$) - (MPa-s)
n_s = journal speed (rps)
p = unit bearing pressure i.e. load per unit of projected area (N/mm^2)

ϕ = angle of eccentricity or attitude angle (it locates the position of minimum film thickness with respect to the direcⁿ of load)

- The coefficient of friction variable (CFV) is given by

$$(CFV) = (r/c) \cdot f$$

f = coeff. of friction.

- The frictional torque is given by

$$(M_t)_f = f W . r \quad N mm.$$

$$\text{Frictional Power} = (2\pi n_s)(fWr) \quad N\text{-}mm/s$$
$$= (2\pi n_s)(fWr) \times 10^{-3} \ W$$
$$= (2\pi n_s)(fWr) \times 10^{-6} \ kW$$

$$(kW)_f = \frac{2\pi n_s f . W r}{10^6}$$

The flow variable is given by

$$FV = \frac{q}{rcn_s l}$$

l = length of bearing (mm)
q = flow of lubricant (mm^3/s)

* Temperature Rise :-

- Heat is generated in the bearing due to the viscosity of lubricating oil.
- The frictional work is converted into heat, which increases the temp. of lubricant.
- Assuming that the total heat generated in the bearing is carried away by the total oil flow in the bearing.
- Expression for temp. rise can be given as

$$(kW)_f = (2\pi n_s)(fWr) \times 10^{-6}$$

- The heat generated is given by

$$H_g = (kW)_f = (2\pi n_s)(fWr) \times 10^{-6} \quad kW \text{ or } kJ/s$$

substituting $f = (c/r)\, CFV$ & $W = 2plr$ in the above equation gives

$$H_g = (2\pi n_s)\left(\frac{c}{\cancel{r}}\right) CFV \cdot (2plr)\, \cancel{r} \times 10^{-6}$$

$$H_g = (4\pi) \times 10^{-6} \times r c\, n_s \cdot lp\, (CFV) \quad \text{—— (1)}$$

- The heat carried away (H_c) by oil flow is given by

$$H_c = m\, C_p \cdot \Delta t$$

where m = mass of lubricating oil passing through bearing kg/s

C_p = specific heat of lubricating oil (kJ/kg°C)

Δt = temp rise ($^\circ C$)

— The mass 'm' of lubricating oil is given by

$$m = \rho \cdot q \, (10^{-6}) \text{ kg/s.}$$

But $q = (rc\,n_s\,l)\,f\,V.$ → substituting in above eqn gives m

$$m = \rho\,(rc\,n_s\,l)\,f\,V \times 10^{-6} \text{ kg/s} \quad \text{—— (3)}$$

substituting (3) in (2)

$$H_c = C_p \cdot \Delta t \; \rho\,(rc\,n_s\,l)\,f\,V \times 10^{-6} \quad \text{—— (4)}$$

— Equating H_g & H_c gives

$$\Delta t = \left(\frac{4\pi P}{\rho\,C_p}\right)\left(\frac{CFV}{fV}\right) \quad \text{—— (5)}$$

for most lubricating oil $\rho = 0.86$
$C_p = 1.76$ kJ/kg$^\circ$C

substituting these in (5) gives

$$\Delta t = \frac{8.3P\,(CFV)}{fV}$$

— Avererage temp. of lubricating oil is given by

$$T_{ave} = T_i + \left(\frac{\Delta t}{2}\right)$$

T_i = inlet temp.

~~Bearing~~ Design - Selection of Parameters:-

- In priliminary stages of journal bearing design, it is required to select suitable values for the following parameters
  - i) length to diameter ratio (L/d) ratio
  - ii) unit bearing pressure (p)
  - iii) start up load
  - iv) radial clearance (c)
  - v) minimum oil film thickness (h_o) &
  - vi) Maximum oil film ~~thick~~ temperature

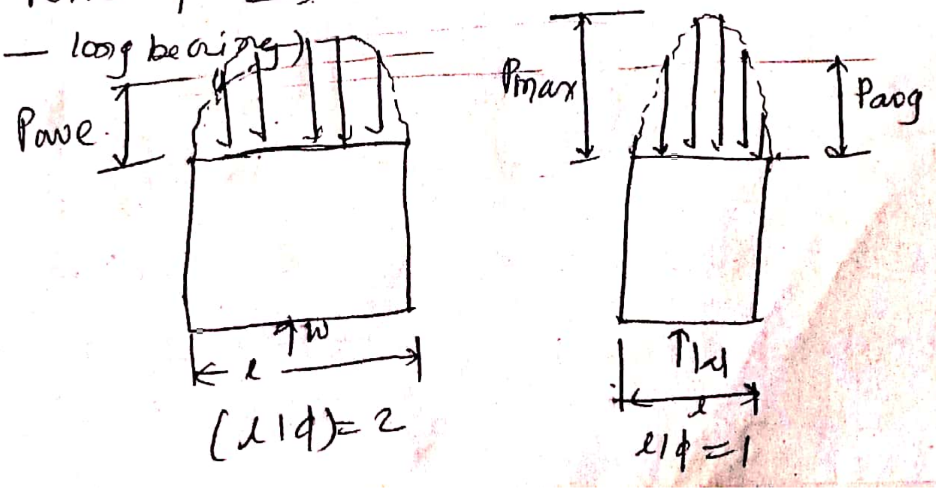
(I) length to diameter (l/d) ratio :-

a) l/d more than 1 → long bearing
b) l/d less than 1 → short bearing
c) l/d = 1 → square bearing

→ dia. of shaft is determined by strength or rigidity consideration & not on bearing capacity.

- shaft design based on permissible stress, permissible angle of twist or permissible lateral deflection.
- so the bearing length is the parameter that designer can decide to obtain given bearing capacity.
- (l/d) ratio affects bearing performance.
- As (l/d) ratio ↑ ⇒ resulting film pressure ↑.

(length more — long bearing)



- As $(l/d) \uparrow \Rightarrow p \uparrow$.
So long bearing has more load carrying capacity compared to short bearing.
- short bearing has greater side flow which improves heat dissipation.
- long bearings are more susceptible to metal to metal contact at two edges, when shaft is deflected under load.
- longer the bearing, the more difficult it is to get sufficient oil flow through passage between journal & bearing.

→ so preferably $(l/d) = 1$ or less than 1

- When shaft & bearing are precisely aligned, shaft deflect within limit and cooling of lubricant & bearing does not present a serious problem.
- (l/d) ratio $= 0.5$ to 2 (in practice)

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(II) Bearing Unit Pressure :- p (N/mm^2)

- load per unit of project area of the bearing in running condition.

- It depends upon bearing material, operating temp, nature & frequency of load & service condition.

Permissible Bearing Pressure p (N/mm^2)

I) Diesel Engine	Main bearing	— (5-10)
	crank pin	— (7 to 14)
	Gudgeon pin	— (13-14)
II) Automotive Engine	Main bearing	(3-4)
	crank pin	(10-14)
III) Air compressors	Main bearing	(1-1.5)
	crank pin	(1.5-3)
IV) Centrifugal pump	Main Bearing	(0.5-0.7)
V) Electric Motor	Main Bearing	(0.7-1.5)
VI) Transmission shaft	light duty	→ 0.15
	Heavy duty	→ 1.0
VII) Machine Tools	Main Bearing	→ 2

(III) Start-up load :-

- Unit bearing pressure for starting condition should not exceeds $2\ N/mm^2$.

- start-up load is static load when shaft is stationary. (ie. dead wt. of shaft & attachment)

- startup load can be used to determine min. length of bearing on the basis of starting condition.

(iv) Radial clearance :- (c)

- Radial clearance should be small to provide necessary velocity gradient.
- This requires costly finishing operation, rigid mounting of bearing assembly & clean lubricating oil without any foreign particles.
- This increases initial & maintenance cost.
- Practical value for c = 0.001 mm per mm of journal radius.

$$c = (0.001)r$$

- Practical values for c for commonly used bearing materials are

Babbits	$(0.001)r$	$(0.00167)r$
copper lead	$(0.001)r$	$(0.001)r$
Aluminium Alloy	$(0.002)r$	$(0.0025)r$

(v) Minimum oil film thickness :-

- The surface finish of journal & bearing is governed by value of minimum oil film thickness selected by ~~the~~ designer.
- There is lower limit for minimum oil film thickness below which metal-to metal contact occurs & hydrodynamic film breaks

$$h_0 = (0.0002)r$$

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(vi) Maximum oil film Temperature :-

- lubricating oil tends to oxidise when operating temp. exceeds 120°. Also surface of babbit bearing tends to soften at 125°C (for $p = 7\ N/mm^2$) & at 190°C ($p = 1.4\ N/mm^2$)

- operating temp. should be kept within these limits
- limiting temp = 90°C for bearing made of babbit

(Babbitt → alloy of tin, antimony, copper, lead)

* Bearings can be designed for two different conditions

i) Bearing for max. load carrying capacity

ii) Bearing for min. frictional losses.

* Optimum value of (h_0/c) for full journal Bearing

(l/d) ratio.	(h_0/c) for max. load	(h_0/c) for min. friction
∞	0.66	0.6
1	0.53	0.3
0.5	0.43	0.12
0.25	0.27	0.03

Example (1)

Following data is given for a 360° hydrodynamic bearing

Radial load = 3.2 kN

Journal speed = 1490 rpm

Journal dia. = 50 mm

Bearing length = 50 mm

Radial Clearance = 0.05 mm

viscosity of lubricant = 25 cP

Assuming that the total heat generated in bearing is carried by the total oil flow in the bearing. calculate

i) coeff. of friction

ii) power lost in friction

iii) minimum oil film thickness

iv) flow requirement in lit/min.

v) temp rise.

→ $P = W/ld = \dfrac{3.2 \times 10^3}{50 \times 50} = 1.28\ N/mm^2$

$$S = \left(\frac{r}{c}\right)^2 \cdot \left(\frac{\mu n_s}{P}\right) = \left(\frac{50}{0.05}\right)^2 \left[\frac{25 \times 10^{-9} \times (1490/60)}{1.28}\right]$$

$S = 0.121$

$l/d = 1$

From table

$CFV = \left(\frac{r}{c}\right) f = 3.22 \quad \left(\frac{h_o}{c}\right) = 0.4 \quad \frac{Q}{n_s r c l} = 4.33$

so $f = 3.22\left(\frac{c}{r}\right) = 3.22\left(\frac{0.05}{25}\right) = 0.00644$

$h_o = 0.4\ c = 0.4(0.05) = 0.02\ mm$

$Q = 4.33\ n_s r c l = 4.33(25)(0.05)(1490/60)(50) =$

$$Q = 6720.5 \ mm^3/s$$ $1000 \ cm^3 = 1 \ lit$

$$= 6720.5 \times 10^{-3} \ cm^3/s$$

$$= 6720.5 \times 10^{-3} \times 10^{-3} \ lit/s$$

$$\boxed{Q = 6720.5 \times 10^{-6} \times 60 \ lit/min}$$

$$(kW)_f = \frac{2\pi n_s f W r}{10^6}$$

$$= \frac{2\pi (1490/60)(0.00644)(3.2)(1000)(25)}{10^6}$$

$$= 0.08$$

temp. rise

$$\Delta t = 8.3 P\left(\frac{CFV}{FV}\right)$$

$$= \frac{8.3 \times 1.28 \times 3.22}{4.33}$$

$$= 7.9^\circ C.$$

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Example ②

Following data is given for a full hydrodynamic bearing used for electric motor:-

radial load = 1200 N

Journal speed = 1440 rpm

Journal diameter = 50 mm

static load on bearing = 350 N

The values of surface roughness (cla) of the journal and the bearing are 2 & 1 micron respectively. The minimum oil film thickness should be five times the sum of surface roughness of the journal and the bearings. Determine:

i) length of the bearing

ii) radial clearance

iii) minimum oil film thickness

iv) viscosity of oil lubricant

select suitable oil for this application assuming the operating temp as 65°C

→ $W = 1200\,N$, $d = 50\,mm$, $n_s = 1440\,rpm$,

$W_{static} = 350\,N$.

minimum oil film thickness $h_0 = 5(2+1)$

— Static load on bearing is 350 N & corresponding start up bearing pressure is $2\,N/mm^2$ (Assume)

$$p = \frac{W}{ld} \quad l = \frac{W}{pd} = \frac{350}{2 \times 50} = 3.5\,mm \quad \text{—— ①}$$

— Radial load acting on bearing is $W_R = 1200\,N$ & for electric motor, unit bearing pressure is 0.7 to 1.5 N/mm^2

Assuming $p = 1\,N/mm^2$

$$l = \frac{W}{pd} = \frac{1200}{1 \times 50} = 24\,mm \quad \text{—— ②}$$

From ① & ②, $l = 24\,mm$.

$(l/d) = (24/50) = 0.48 \simeq 0.5$

We assume standard value of $(l/d) = 0.5$.

$l = 0.5\,d = 0.5(50) = 25$ mm

$p = \frac{W}{l\,d} = \frac{1200}{25(50)} = 0.96\ N/mm^2$

– Radial clearance.

$c = (0.001)r = 0.001 \times 0.25 = 0.025$ mm

– Minimum oil film thickness

$h_o = 5(2+1)\ mm/1000 = 0.015$ mm.

from table $(l/d) = 0.5$ & $(h_o/c) = 0.6$

table → $S = 0.779$

$\frac{q}{r c n_s l} = 4.29$

$0.779 = S = \left(\frac{r}{c}\right)^2 \times \left(\frac{\mu n_s}{P}\right) = \left(\frac{25}{0.025}\right)^2 \times \frac{\mu(1440/60)}{0.96}$

$\mu = 31.16 \times 10^{-9}\ Ns/mm^2 = cP.$

from table at $\mu = 31.16$ cP & at $65^{\circ}C$

SAE-40 oil is suitable for this applicatio

$q = 4.29\ r c n_s l = 4.29\ (25)(0.025)(24)(25)$

$q = 1608.75\ mm^3/s$

Example (03).

Design a full Hydrodynamic bearing with following specification for m/c tool application :-

Journal diameter = 75 mm

Radial load = 10 kN

Journal speed = 1440 rpm

min. oil film thickness = 22.5 microns

inlet temperature = 40°C

Bearing Material = Babbitt

Determine the length of the bearing & select suitable oil for this application.

→ $W = 10\times10^3$ N, $d = 75$ mm, $n_s = (1440/60) = 24$ rps.

Permissible bearing pressure for m/c tool = 2 N/mm² at running condition.

$$p = \frac{W}{ld} \quad \therefore \; l = \frac{W}{pd} = \frac{10\times10^3}{2\times75} = 66.67 \text{ mm}.$$

$$l/d = \frac{66.67}{75} = 0.89$$

Assuming standard value for (l/d) ratio as 1

$$(l/d) = 1$$

$$l = d = 75 \text{ mm}.$$

— Radial clearance for babbit material

$$c = (0.001)\, r = (0.001)(75/2) = 0.0375 \text{ mm}$$

$$h_o = 22.5\times10^{-3} = 0.0225 \text{ mm}$$

— from table. $(l/d) = 1$, $(h_o/c) = 0.6$.

so $S = 0.264$ $\left(\frac{r}{c}\right) f = 5.79$, $\frac{Q}{r c n_s l} = 3.99$

$$0.264 = S = \left(\frac{r}{c}\right)^2\left(\frac{\mu n_s}{P}\right) = \left(\frac{75}{0.0375}\right)^2 \times \frac{\mu(.24)}{(1.78)}$$

$$\mu = 19.58 \times 10^{-9} \; Ns/mm^2$$

$$= 19.58 \; cP.$$

<u>Rise temp</u>

$$\Delta t = \frac{8.3P}{}\left(\frac{CFV}{FV}\right) = \frac{8.3 \times 1.78 \times 5.79}{3.99}$$

$$\Delta t = 21.44^\circ C$$

$$T_{avg} = T_i + \frac{\Delta t}{2} = 40 + \frac{21.44}{2} = 50.72^\circ C$$

for temp. $50.72^\circ C$ & $\mu = 19.58$ cP.
SAE-10 oil is best suitable.

Example: The following data is given for a 360° hydrodynamic bearing:-

length to diameter ratio = 1

Journal speed = 1350 rpm

journal diameter = 100 mm

diametral clearance = 100 μm

external load = 10 kN

The value of minimum film thickness variable is 0.3. find the viscosity of oil that need be used.

→ $W = 9\ kN$, $n = 1350$ rpm, $d = 100$ mm, diametral clearance $= 100\ \mu m$

$l/d = 1$ $h_0/c = 0.3$

Summerfeld no. $(l/d) = 1$ & $(h_0/c) = 0.3$

$$S = \frac{0.0446 + 0.121}{2} = 0.0828$$

$$P = \frac{W}{ld} = \frac{9000}{100 \times 100} = 0.9\ N/mm^2$$

$$c = \frac{1}{2}(\text{diametral clearance}) = \frac{1}{2}(100 \times 10^{-3})\ mm$$

$$= 50 \times 10^{-3}\ mm$$

$$S = (r/c)^2 \frac{\mu n_s}{P} = \left(\frac{50}{50 \times 10^{-3}}\right)^2 \frac{\mu \times (1350/60)}{0.9} = 0.0828$$

$$\mu = 3.312 \times 10^{-9}\ Ns/mm^2 = 3.312\ cP.$$

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Example :- The following data is given for a 360° hydrodynamic bearing :-

radial load = 10 kN
Journal speed = 1440 rpm
unit bearing pressure = 1000 kPa
Clearance ratio (r/c) = 800
viscosity of lubricant = 30 mPa s

Assuming that the total heat generated in the bearing is carried by the total oil flow in the bearing. Calculate

i) Dimensions of the bearings
ii) coefficient of ~~bearing~~ friction
iii) power lost in friction
iv) total flow of oil
v) side leakage &
vi) temperature rise

→ $W = 10$ kN, $n = 1440$ rpm, $P = 1000$ kPa, $(r/c) = 800$,
$\mu = 30$ mPa s

Bearing dimensions :-

Assuming $(l/d) = 1$

$P = 1000 \times 10^{-3}$ Pa $= 1$ MPa

$P = \frac{W}{ld}$ $\quad l^2 = \frac{W}{P}$ $\Rightarrow$ $l = 100$ mm

$l/d = 1 \quad l = d = 100$ mm

→ $\mu = 30$ mPa s $= 30 \times 10^{-3}$ Pa s $= 30 \times 10^{-9}$ MPa s

$S = (r/c)^2 \frac{\mu n_s}{P} = (800)^2 \frac{30 \times 10^{-9} \times (1440/60)}{1}$

$S = 0.4608$

By linear interpolation

$(r/c) f = 5.79 + (12.8 - 5.79)\left(\frac{0.4608 - 2.64}{0.631 - 0.264}\right)$

$= 9.55$

$$\frac{Q}{rcn_s l} = 3.99 - (3.99 - 3.59)\left[\frac{0.4608 - 0.264}{0.631 - 0.264}\right]$$

$$= 3.78$$

$$\frac{Q_s}{Q} = 0.497 - (0.497 - 0.28)\left[\frac{0.4608 - 0.264}{0.631 - 0.264}\right]$$

$$= 0.38$$

– $\phi = 9.55\,(c/r) = 0.0119$

– $$(kW)_f = \frac{2\pi n_s f W r}{10^6}$$

$$= \frac{2\pi\,(1440/60)\,(0.0119)\,(10000)\,(50)}{10^6}$$

$$= 0.9$$

– Total flow of oil

$$Q = 3.78\, rcn_s l$$

$$= 3.78\,(50)\left(\frac{50}{8000}\right)(1440/60)\,(100)$$

$$= 28350 \text{ mm}^3/s$$

– side leakage

$$Q_s = 0.38\,Q = 10773 \text{ mm}^3/s$$

– Temp. rise

$$\Delta t = \frac{8.3\,P\,(CFV)}{(FV)}$$

$$= \frac{8.3 \times 1 \times 9.55}{3.78}$$

$$= 22.97\,^\circ C$$

Example 1:- The following data is given for a 360° hydrodynamic bearing

radial load = 6.5 kN
journal speed = 1200 rpm
journal dia = 60 mm
bearing length = 60 mm
min. oil film thickness = 0.009 mm

The class of fit is H7e7 (fine) normal running fit specify the viscosity of the lubricating oil that you will recommend for this application.

— $W = 6.5$ kN, $n = 1200$ rpm, $d = 60$ mm, $l = 60$ mm,
$h_0 = 0.009$ mm

– $S =$

for hole & shaft of H7e7 running fit

Hole limits (60 + 0.00) & (60 + 0.03) mm

Shaft limits (60 − 0.09) & (60 − 0.06) mm

Average dia. of bearing & journal would be 60.015 & 59.925 mm respt

$$c = \frac{1}{2}(60.015 - 59.925) = 0.045 \text{ mm}$$

$$\left.\begin{array}{l} (h_0/c) = \dfrac{0.009}{0.045} = 0.2 \\ (l/d) = (60/60) = 1 \end{array}\right\} \quad S = 0.0446$$

$$\mu = S\left(\frac{c}{r}\right)^2 \frac{P}{n_s} = 0.0446\left(\frac{0.045}{30}\right)^2\left(\frac{6500}{60 \times 60}\right)\frac{1}{1200 \times 60}$$

$$\boxed{\mu = 9.06 \text{ cP}}$$

Example :-

following data is given for a 360° hydrodynamic bearing

radial load = 2 kN

Journal diameter = 50 mm

Bearing length = 50 mm

Viscosity of oil = 20 mPa s

Specify radial clearance that need to be provided so that when the journal is rotating at 2800 rpm, the minimum film thickness is 30 microns. Evaluate the corresponding coeff. of friction.

→ $W = 2000 N$, $d = 50$ mm, $L = 50$ mm, $\mu = 20$ mPa s $= 20 \times 10^{-9}$ MPa s.

$n = 2800$ rpm

$h_o = 30 \times 10^{-3}$ mm

$$p = \frac{W}{ld} = \frac{2000}{50 \times 50} = 0.8 \ N/mm^2$$

$$S = \left(\frac{r}{c}\right)^2 \left(\frac{\mu n_s}{p}\right) = \left(\frac{25}{c}\right)^2 \frac{20 \times 10^{-9} \times (2800/60)}{0.8}$$

$$c = \sqrt{\frac{0.73 \times 10^{-3}}{S}}$$

S & c are unknown

Use trial & Error

Trial ① Assume $(h_o/c) = 0.9$ → $S = 1.33$

$C = h_o/0.9 = 0.0333$ mm

$$C = \sqrt{\frac{0.73 \times 10^{-3}}{S}} = 0.0234 \text{ mm}$$

calculated = $c = 0.0234$ & c by realation $c = 0.0333$

Trial ② Assume $h_o/c = 0.8$ → $S = 0.631$

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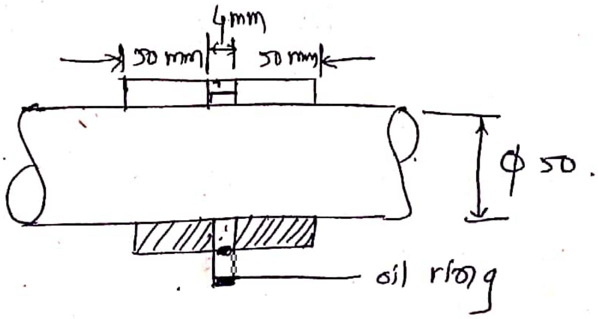
Example :-

An oil ring bearing of a transmission shaft is shown in Fig. There is no hydrodynamic action over the width of 4 mm of the oil ring. The total radial load acting on the journal is 20 kN and the journal rotates at 1450 rpm. The radial clearance and minimum film thickness are 20 and 5 micron respectively. Calculate.

i) Viscosity of the lubricant and

ii) required quantity of oil.



oil ring divides bearing into two half & each treated as separate hydrodynamic bearing carrying a load of (20/2) or 10 kN.

$$(l/d) = (50/50) = 1$$

$$(h_0/c) = (5/20) = 0.25$$

from Table

$$S = 0.0637$$

$$\left(\frac{Q}{r c n_s l}\right) = 4.5975$$

$$p = \frac{W}{l d} = 4 \ N/mm^2$$

$$\mu = S\left(\frac{c}{r}\right)^2\left(\frac{p}{n_s}\right)$$

$$= 6.7478 \times 10^{-9} \ Ns/mm^2$$

$$Q = 4.5975 \ r c n_s l = 2747 \ mm^3/s$$

$$Q_{total} = 2Q = 5494 \ mm^3/s = 5494 \times 60 \times 10^{-6} \ lit/min = 0.33 \ lit/m$$

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A 360° Hydrodynamic journal bearing has 50 mm diameter and 50 mm length. the journal is carrying a load of 15 kN and rotating at a speed of 1440 rpm. The eccentricity ratio is 0.75. If the radial clearance is 20 microns calculate.

i) the minimum oil-film thickness;

ii) the viscosity of oil;

iii) the quantity of oil in circulation

iv) the oil leakage through sides

→ $d = 50$ mm

$l = 50$ mm

$W = 15000$ N

$n_s = 1450/60 = 24.17$ rps

$\varepsilon = 0.75$

$c = 0.02$ mm

$l/d = 1$

$$\frac{h_o}{c} = 1 - \varepsilon = 1 - 0.75 = 0.25$$

$h_o = 5 \times 10^{-3}$ mm $= 5$ μm

$$\boxed{(h_o/c) = 0.25}$$

From table

$h_o/c = 0.2$

(h_o/c) 0.25 →

$h_o/c = 0.4$

ii) Viscosity of lubricant.

$p = W/ld = 6$ N/mm²

$S = 0.121$

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu n_s}{p}$$

$\mu = 4.499 \times 10^{-9}$ Ns/mm²

Total flow of lubricant

$$\frac{Q}{r c n_s l} = 4.5475$$

$$Q = \cancel{0.1648}\ 2767.486\ \cancel{m^3/s} \times 10 \ mm^3/s.$$

$$Q = 0.1648 \ lit/min.$$

Bearing failure :- Causes & Remedies

- insufficient lubricant
- contamination of lubricant
- faulty assembly

① Abrasive Wear :-

- Scratches in dirⁿ of motion due to embeded particles or contamination.

Remedy - proper enclosures, cleanliness of lubricating oil & use of high μ oil

② Wiping of Bearing Surface :-

- excessive rubbing in bearing surfaces. result in melting & smearing
- inadequate clearance, excessive transient load & insufficient oil supply

③ Corrosion :-

- chemical attack of reactive agent
- oxidation product corrode material such as lead, copper, cadmium, zin
- lead react rapidly with oxidation agent

Remedy :- use of oxidation inhibitors as additives in lubricating oil

④ Distortion :-

- misalignment or incorrect type of fit
- too tight fit - distort bore
- foreign particle trapped betⁿ bearing & housing

Remedy :- correct selection of fit, proper assembly